A Hybrid Dielectric Slab-Beam Waveguide for the Sub-Millimeter Wave Region

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***Abstract*—A hybrid dielectric slab-beam waveguide is suggested which should be well suited as transmission medium for the design of planar quasi-optical integrated circuits and devices operating in the mm and sub-mm wave regions. The new guide consists of a grounded dielectric slab into which a sequence of equally spaced cylindrical lenses is fabricated. (The center line of the slab guide is the axis of the lenses). The structure uses two distinct wave guiding principles in conjunction with each other to guide electromagnetic waves. In the direction normal to the slab surface, the guided fields behave as surface waves of the slab guide; their energy is largely confined to the interior of the dielectric and they are guided by total reflection at the slab surface. In the lateral direction the waves behave as Gauss–Hermite beammodes that are guided by the lenses which periodically reconstitute their cross sectional phase distribution, resulting in a wave beam that is iterated with the lens spacing. The guided fields are in effect TE and TM modes.**

The analysis of the new guiding structure is presented: The mode spectrum is calculated and the iteration loss due to the finite size of the lenses is estimated.

I. Introduction

RECTANGULAR dielectric waveguide and its variants such as image guide, insular guide, trapped image line and suspended dielectric line are well suited for use in the mm-wave region. Their low loss gives them an advantage over microstrip line and they are easier and less expensive to fabricate than metal waveguide. A review of these guides may be found for example in [1], [2] which also present bibliographies on this subject.

As is well known, the guidance principle employed in dielectric guides is total reflection at the dielectric surfaces, which confines the transmitted energy in effect to the interior of the guides. Typically, the width of these guides is chosen to be somewhat less than a half wavelength in the guide material to avoid over-moding. A consequence is that at the small wavelengths in the upper mm-wave region the guide width becomes extremely narrow, in particular when high-ε material is used and the guides would be very difficult to fabricate.

The hybrid dielectric slab-beam waveguide suggested in this paper resolves this problem by the use of a quasi-optical guidance principle (iteration by periodic refocusing) to provide beam confinement in the lateral direction; this permits one to make the width of the guide electrically large. The guide will propagate a spectrum of modes. But the guidance principle employed here insures that the field distribution of the modes is virtually independent of the guide width. In other words, if a single mode is launched on the guide it will suffer little degradation due to mode conversion as it travels down the line, even if the guide width shows some variation. Hence, there is no need for maintaining a constant width at tight tolerances. In addition, bends and transitions are easily implemented in this guide in standard quasi-optical technology while causing minimum radiation loss and mode conversion; and guide sections operated as open resonators should be well suited for the design of quasi-optical power combiners that could serve as single mode power sources for these guides [3].

The configuration of the new guide is shown in Fig. 1. It consists of a thin grounded dielectric slab of rectangular cross section, into which a sequence of equally spaced cylindrical lenses has been fabricated. As indicated in Fig. 1, the axis of the lenses coincides with the center line of the slab guide (propagation direction of the guide). The spacing of the lenses s is assumed to be in the order of many guide wavelengths λ; the width of the slabguide w is in the order of at least several λ; and the thickness d of the guide typically will be chosen sufficiently small so that only the fundamental surface wave mode can exist on the slab. The convex shape of the lenses indicated in Fig. 1(a) applies to the case that the permittivity of the lenses exceeds that of the guide; in the opposite case, the lenses will have the concave shape shown in Fig. 1(b), which may simplify their fabrication and reduce their diffraction losses.

The structure uses two distinct waveguiding principles in conjunction with each other to confine and guide electromagnetic waves. In the x-direction of Fig. 1, the field distribution of a guided wave is that of a surface-wave mode of the slabguide; the wave is guided by total reflection at the dielectric-to-air interface and its energy is transmitted primarily within the dielectric. In the y-direction, the field distribution is that of a Gaussian (or Gauss–Hermite) beammode which is guided by the lenses through periodic reconstitution of the cross sectional phase distribution, resulting in an "iterative wavebeam" whose period is the spacing of the lenses. The guided modes are, in effect, TE- or TM-polarized with respect to the z-direction, the propagation direction of the guide.

The waveguide should be useful in particular for the sub-mm region of the electromagnetic spectrum. It bridges the gap between conventional dielectric waveguides employed in the mm-wave region and slab type dielectric waveguides used at optical wavelengths. Combining structural simplicity, approaching that of a slabguide, with the increased lateral

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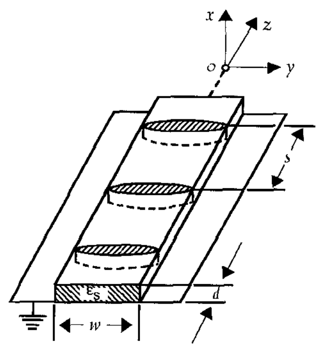
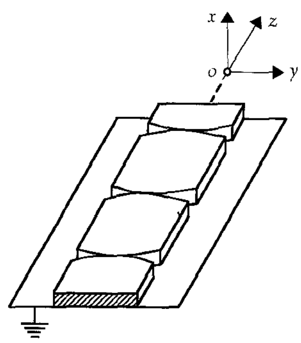


(a)



(b)

Fig. 1. Dielectric slab-beam waveguide. The lenses embedded in the slab provide periodic refocusing and have a convex shape, case (a), if $\varepsilon_{\text{lens}} > \varepsilon_{\text{slab}}$. If $\varepsilon_{\text{lens}} < \varepsilon_{\text{slab}}$, the lens shape is concave, case (b).

dimensions of quasi-optical devices, it should be easy to fabricate and show good electrical performance. The new guide should be well suited in particular as basic transmission medium for the design of planar integrated circuits and components.

The study of waveguides specifically designed for the submillimeter wave region is a relatively new research area. Reference [4] provides an excellent, detailed review on (other) monolithic guides that are promising candidates.

II. Theory of Dielectric Slab-Beam Waveguide of Infinite Cross Section

We first treat the idealized dielectric slab-beam waveguide of Fig. 2. It consists of a grounded dielectric slab of permittivity ε_s, which in the y-direction extends to infinity. In the planes $z = (2\mu - 1)z_t$, with $\mu = 0, 1, 2 \cdots$, planar phase transformers are inserted in the guide. The phase transformers extend to infinity, both in the x- and y-directions, and similar to cylindrical lenses introduce a phase shift in the transmitted fields that is quadratic in y and uniform in x. All (active) sources are located in the half space $z < -z_t$.

We formulate the field in the space range $-z_t < z < +z_t$. Since the guide structure in this region is uniform in y and z, but layered in x, it is convenient to write this field as a superposition of an E-field with $H_x \equiv 0$ and an H-field with $E_x \equiv 0$. The E-field and H-field are derived, respectively, from an x-directed electric and magnetic vector potential $\Psi\vec{e}_x$

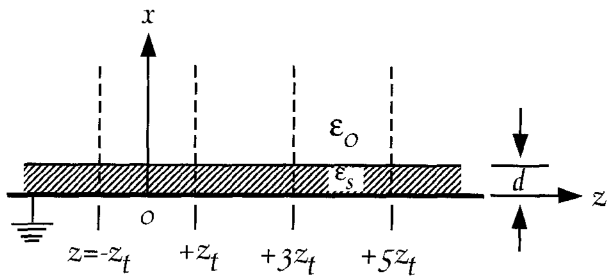


Fig. 2. Idealized dielectric slab-beam waveguide with planar, infinitely thin phase transformers. The slab is assumed to be unbound in the y-direction, and the phase transformers extend to infinity in both the x- and y-directions.

and $\Phi\vec{e}_x$. The governing equation are, for the E-field:

$$\vec{E} = \left(\frac{k_0}{k}\right)^2 \vec{\nabla}\times\left(\vec{\nabla}\times\Psi\vec{e}_x\right), \quad \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\vec{H} = jk_0\left(\vec{\nabla}\times\Psi\vec{e}_x\right) \tag{1}$$

$$\nabla^2\Psi + k^2\Psi = 0 \tag{2a}$$

$$\text{with } \Psi,\ \left(\frac{k_0}{k}\right)^2\frac{\partial\Psi}{\partial x} \quad \text{continuous at } x = d \tag{2b}$$

$$\frac{\partial\Psi}{\partial x} = 0 \quad \text{at } x = 0 \tag{2c}$$

and for the H-field:

$$\vec{E} = -jk_0\left(\vec{\nabla}\times\Phi\vec{e}_x\right), \qquad \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\vec{H} = \vec{\nabla}\times\left(\vec{\nabla}\times\Phi\vec{e}_x\right) \tag{3}$$

$$\nabla^2\Phi + k^2\Phi = 0 \tag{4a}$$

$$\text{with } \Phi,\ \frac{\partial\Phi}{\partial x} \quad \text{continuous at } x = d \tag{4b}$$

$$\Phi = 0 \qquad \text{at } x = 0 \tag{4c}$$

Both Ψ and Φ satisfy an appropriate radiation condition for $x \to \infty$; and k in (1) through (4) is defined as:

$$k = k_s = k_0\sqrt{\varepsilon_s} \quad \text{for } 0 < x < d$$
$$k = k_0 \quad \text{for } d < x < \infty \tag{5}$$

where $k_0 = 2\pi/\lambda_0$ is the free space propagation constant.

We write the potentials Ψ and Φ as superpositions of elementary waves defined by the modes of the grounded dielectric slab guide or, in mathematical terms, by the separated solutions of (2) and (4). It is well known that the spectrum of slab guide modes consists of two parts, a discrete spectrum of surface wave modes guided by the slab and a continuous spectrum of radiative modes (quasi-modes) describing radiation effects. Taken together, these two spectra form a complete orthogonal system into which any field, whose distribution in a plane $z =$ const is known, can be expanded. The fields of interest in the present context are purely bound and do not radiate. Hence, in any guide section between adjacent phase transformers, these fields can be represented solely in terms of the surface wave modes while the spectrum of radiative modes can be disregarded.

In the two-dimensional case, that all field components are independent of the y-coordinate, the slabguide modes are well known [5]–[7]. E-type fields in this case reduce to TM-waves with the components E_x, E_z, H_y and H-type fields to TE-waves with the components E_y, H_x, H_z. The potentials Ψ and Φ are given by

$$\Psi_n(x,z) = F_n(x)e^{-j\beta_n z} \quad n = 0,1\cdots N \qquad \Phi_n(x,z) = G_n(x)e^{-j\overline{\beta}_n z} \quad n = 0,1\cdots \overline{N} \tag{6}$$

with

$$F_n(x) = \cos\left[\left(\sqrt{k_s^2-\beta_n^2}\right)\right]x \quad \text{for } 0 < x < d \tag{7a}$$
$$F_n(x) = \cos\left[\left(\sqrt{k_s^2-\beta_n^2}\right)d\right]e^{-\left(\sqrt{\beta_n^2-k_0^2}\right)(x-d)} \quad \text{for } d < x < \infty$$

and

$$G_n(x) = \sin\left[\left(\sqrt{k_s^2-\overline{\beta}_n^2}\right)\right]x \quad \text{for } 0 < x < d \tag{7b}$$
$$G_n(x) = \sin\left[\left(\sqrt{k_s^2-\overline{\beta}_n^2}\right)d\right]e^{-\left(\sqrt{\overline{\beta}_n^2-k_0^2}\right)(x-d)} \quad \text{for } d < x < \infty$$

The propagation constants β_n, $\overline{\beta}_n$ are determined by the characteristic equations

$$\sqrt{k_s^2-\beta_n^2}\tan\left[\left(\sqrt{k_s^2-\beta_n^2}\right)\right]d = \left(\frac{k_s}{k_0}\right)^2\sqrt{\beta_n^2-k_0^2} \quad \text{for TM polarization } (\Psi_n) \tag{8a}$$

and

$$\sqrt{\overline{\beta}_s^2-k_0^2}\tan\left[\left(\sqrt{k_s^2-\overline{\beta}_n^2}\right)\right]d = -\sqrt{k_s^2-\overline{\beta}_n^2} \quad \text{for TE polarization } (\Phi_n) \tag{8b}$$

respectively. Solution of these equations [5]–[7] yields the well known dispersion curves of the surface-wave modes of the slabguide, i.e. β_n/k_0 and $\overline{\beta}_n/k_0$ as functions of k_0d and ε_s. The propagation constants of these modes are in the slow wave region

$$k_0 < \beta_n,\ \overline{\beta}_n < k_s.$$

and the cut-off frequency of the nth mode is given by

$$k_0d = \frac{n\pi}{\sqrt{\varepsilon_s-1}} \quad \text{for TM polarization} \tag{9a}$$
$$k_0d = \frac{(n+1/2)\pi}{\sqrt{\varepsilon_s-1}}. \quad \text{for TE polarization} \tag{9b}$$

The total number of surface wave modes supported by a guide of given permittivity and thickness is equal to the largest integer satisfying the condition

$$N < \frac{k_0d}{\pi}\sqrt{\varepsilon_s-1} \quad \text{for TM polarization} \tag{10a}$$
$$\overline{N} < \frac{k_0d}{\pi}\sqrt{\varepsilon_s-1}-1/2. \quad \text{for TE polarization} \tag{10b}$$

For later use we note that the functions $F_n(x)$, $G_n(x)$ satisfy the orthogonality relations:

$$\int_{x=0}^{\infty}\left(\frac{k_0}{k}\right)^2 F_n(x)F_{n'}(x)\,dx = \frac{1}{2k_0\varepsilon_s}\cdot\left[k_0d + \frac{\varepsilon_s}{\sqrt{\left(\frac{\beta_n}{k_0}\right)^2-1}\left[\left(\frac{\beta_n}{k_0}\right)^2(\varepsilon_s+1)-\varepsilon_s\right]}\right]\delta_{nn'} \tag{11a}$$

$$\int_{x=0}^{\infty} G_n(x)G_{n'}(x)\,dx = \frac{1}{2k_0}\cdot\left[k_0d + \frac{1}{\sqrt{\left(\frac{\beta_n}{k_0}\right)^2-1}}\right]\delta_{nn'} \tag{11b}$$

with

$$k = k_s \quad \text{for } 0 < x < d \qquad \delta_{nn'} = 1 \quad \text{for } n' = n$$
$$k = k_0 \quad \text{for } d < x < \infty \qquad \delta_{nn'} = 0 \quad \text{for } n' \neq n$$

Generalization to the three-dimensional case, where the fields transmitted by the guide may depend on the y-coordinate as well, is straight forward. The slabguide modes in this case remain separated solutions of (2) and (4), and since the guide structure is uniform in the y-direction, the y-dependence of these modes is of the form e^{jvy} with $-\infty < v < +\infty$. The three-dimensional surface wave modes thus take the form

$$\Psi_n(x,y,z) = F_n(x)e^{j(vy-h_nz)} \quad n = 0,1\ldots N \tag{12a}$$
$$\Phi_n(x,y,z) = G_n(x)e^{j(vy-\overline{h}_nz)} \quad n = 0,1\ldots \overline{N} \tag{12b}$$

with

$$h_n^2 = \beta_n^2 - v^2 \quad \text{and} \quad \overline{h}_n^2 = \overline{\beta}_n^2 - v^2. \tag{12c}$$

Note that the x-dependence of these modes is given by the same functions F_n and G_n as in the two-dimensional case. But, for sufficiently large v, the modes become evanescent in the z-direction; see (12c). The three-dimensional slabguide modes of the *propagating* type are obtained simply by allowing the corresponding two-dimensional modes to propagate in any direction within the y,z-plane, instead of confining them to propagation in the z-direction only. The relationship of the three-dimensional modes of the *evanescent* type to the two-dimensional modes has to be understood in terms of complex directions of propagation, i.e., in the more formal way of (12c).

With (12), any field guided by the structure of Fig. 2 can be written as the sum of an E-type field

$$\Psi(x,y,z) = \sum_{n=0}^{N} F_n(x)\int_{-\infty}^{\infty} a_n(v)e^{j(vy-h_nz)}\,dv \tag{13a}$$

$$h_n = \sqrt{\beta_n^2-v^2} \quad \text{for } |v| < \beta_n$$
$$h_n = -j\sqrt{v^2-\beta_n^2} \quad \text{for } |v| > \beta_n \tag{13b}$$

and an H-type field

$$\Phi(x,y,z) = \sum_{n=0}^{\overline{N}} G_n(x)\int_{-\infty}^{\infty} b_n(v)e^{j(vy-\overline{h}_nz)}\,dv \tag{14a}$$

$$\overline{h}_n = \sqrt{\overline{\beta}_n^2 - v^2} \quad \text{for } |v| < \overline{\beta}_n$$
$$\overline{h}_n = -j\sqrt{v^2 - \overline{\beta}_n^2} \quad \text{for } |v| > \overline{\beta}_n . \tag{14b}$$

The functions $a_n(v)$ and $b_n(v)$ are the mode amplitude spectra; N and $\overline{N}$ are given by eqs. (10); the signs of h_n and $\overline{h}_n$ are in accordance with the assumption that all sources are located in the half space $z < -z_t$.

Inserting (13) and (14) into (1) and (3) yields the field strength components of the E-type and H-type fields. In writing down these components we introduce the assumption that—with regard to their y-dependence—these fields are strongly collimated about the z-axis, i.e. we introduce the "wavebeam condition" that the amplitude functions a_n and b_n are significantly different from zero only in a small v-range centered about $v = o$:

$$a_n(v), b_n(v) \to 0 \quad \text{for } |v| > v_n$$
$$\text{where } v_n \ll \beta_n, \overline{\beta}_n . \tag{15}$$

The propagation constant h_n may then be approximated by

$$h_n = \beta_n \text{ in amplitude terms}$$
$$h_n = \beta_n - \frac{1}{2}\frac{v^2}{\beta_n} \text{ in phase terms.} \tag{16}$$

A corresponding approximation holds for $\overline{h}_n$. The approximation is valid in the z-range $\beta_n|z| < 4\pi\left(\frac{\beta_n}{v_n}\right)^4$. Since $v_n \ll \beta_n$ this z-range extends over many wavelengths and is assumed here to exceed the range $-z_t < z < +z_t$.

Neglecting higher order terms involving integrals over $v^\mu a_n(v)$ and $v^\mu b_n(v)$, with $\mu = 1, 2\cdots$, the field derived from the electric potential $\Psi\vec{e}_x$ reduces to a TM-wave with the significant components E_x, E_z, H_y:

$$E_x = \left(\frac{k_0}{k}\right)^2 \sum_{n=0}^{N} \beta_n^2 F_n(x) A_n(y,z) e^{-j\beta_n z} \tag{17a}$$
$$E_z = -j\left(\frac{k_0}{k}\right)^2 \sum_{n=0}^{N} \beta_n \frac{dF_n(x)}{dx} A_n(y,z)\, e^{-j\beta_n z} \tag{17b}$$
$$\sqrt{\frac{\mu_0}{\varepsilon_0}}\, H_y = k_0 \sum_{n=0}^{N} \beta_n F_n(x) A_n(y,z) e^{-j\beta_n z} \tag{17c}$$

with

$$A_n(y,z) = \int_{v=-\infty}^{\infty} a_n(v) e^{j\left(vy + \frac{1}{2}\frac{v^2}{\beta_n} z\right)}\, dv \tag{17d}$$

and the field derived from the magnetic potential $\Phi\vec{e}_x$ reduces to a TE-wave with the significant components E_y, H_x, H_z:

$$E_y = -k_0 \sum_{n=0}^{\overline{N}} \overline{\beta}_n G_n(x) B_n(y,z) e^{-j\overline{\beta}_n z} \tag{18a}$$
$$\sqrt{\frac{\mu_0}{\varepsilon_0}}\, H_x = \sum_{n=0}^{\overline{N}} \overline{\beta}_n^2 G_n(x) B_n(y,z) e^{-j\overline{\beta}_n z} \tag{18b}$$
$$\sqrt{\frac{\mu_0}{\varepsilon_0}}\, H_z = j \sum_{n=0}^{\overline{N}} \overline{\beta}_n \frac{dG_n(x)}{dx} B_n(y,z) e^{-j\overline{\beta}_n z} \tag{18c}$$

with

$$B_n(y,z) = \int_{v=-\infty}^{\infty} b_n(v) e^{j\left(vy + \frac{1}{2}\frac{v^2}{\overline{\beta}_n} z\right)}\, dv . \tag{18d}$$

The fields (17) and (18) show the same TM and TE polarizations as the corresponding two-dimensional (y-independent) fields. Although only one of the three "cross polarized" components of each field is identically zero, the wavebeam condition causes the two remaining components (E_y, H_z of the E-field and E_z, H_y of the H-field) to be small so that they can be neglected.

Similar to the theory of conventional beam waveguides [8], [9] the amplitude functions a_n and b_n are expanded into Gauss-Hermite functions

$$a_n(v) = \sum_{m=0}^{\infty} A_{nm} q_m(v) \tag{19}$$
$$b_n(v) = \sum_{m=0}^{\infty} B_{nm} q_m(v)$$

with

$$q_m(v) = (-j)^m He_m\left(\sqrt{2}\frac{v}{v_n}\right) e^{-\frac{1}{2}\left(\frac{v}{v_n}\right)^2}$$

where v_n, the mode parameter, is a constant independent of m and $He_m(u) = (-1)^m e^{u^2/2}(d^m/du^m)(e^{-u^2/2})$. The integrations in (17d) and (18d) can be performed in closed form, with the result:

$$A_n(y,z) = \sum_{m=0}^{\infty} A_{nm} Q_{nm}(y,z) \tag{20a}$$
$$B_n(y,z) = \sum_{m=0}^{\infty} B_{nm} \overline{Q}_{nm}(y,z) \tag{20b}$$

where

$$Q_{nm}(y,z) = Q_m(y,z;v_n,\beta_n) = \sqrt{2\pi}\frac{v_n}{\left[1 + \left(\frac{v_n^2}{\beta_n} z\right)^2\right]^{\frac{1}{4}}} \cdot He_m\left\{\frac{\sqrt{2}\, v_n y}{\left[1 + \left(\frac{v_n^2}{\beta_n} z\right)^2\right]^{\frac{1}{2}}}\right\}$$
$$\otimes \exp\left\{-\frac{1}{2}\frac{v_n^2 y^2}{1 + \left(\frac{v_n^2}{\beta_n} z\right)^2} - j\left[\frac{1}{2}\frac{v_n^4 \frac{z}{\beta_n} y^2}{1 + \left(\frac{v_n^2}{\beta_n} z\right)^2} - \left(m + \frac{1}{2}\right)\tan^{-1}\left(\frac{v_n^2}{\beta_n} z\right)\right]\right\} \tag{21a}$$
$$\overline{Q}_{nm}(y,z) = Q_m(y,z;\overline{v}_n \overline{\beta}_n) . \tag{21b}$$

Thus the total field represented by (17) and (18) can be written as a superposition of the partial fields

$$E_x^{n,m} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{k_0\beta_n}{k^2} H_y^{n,m} = \left(\frac{k_0}{k}\right)^2 F_n(x) Q_{nm}(y,z)$$

$$\cdot e^{-j\beta_n z} \tag{22a}$$

$$E_z^{n,m} = \frac{1}{j\beta_n}\frac{\partial E_x^{n,m}}{\partial x}$$
$$n = 0, 1 \cdots N, \qquad m = 0, 1 \cdots \infty \tag{22b}$$

and

$$E_y^{n,m} = -\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{k_0}{\overline{\beta}_n} H_x^{n,m} = G_n(x)\overline{Q}_{nm}(y,z)e^{-j\overline{\beta}_n z} \tag{23a}$$

$$H_z^{n,m} = \frac{1}{j\overline{\beta}_n}\frac{\partial H_x^{n,m}}{\partial x}$$
$$n = 0, 1 \cdots \overline{N}, \qquad m = 0, 1 \cdots \infty \tag{23b}$$

where the functions F_n and G_n are given by (7) and the propagation constants β_n, $\overline{\beta}_n$ by (8). N and $\overline{N}$ are defined by (10). For any plane $z =$ const., the functions Q_{nm} form a complete system satisfying the orthogonality relation:

$$\int_{-\infty}^{\infty} Q_{nm}(y,z)Q_{nm'}(y,z)dy = 2\pi^{\frac{3}{2}} m!\, v_n \delta_{mm'}. \tag{24}$$

Equations (22) and (23) represent the partial fields in the space range $-z_t < z < +z_t$ of the guiding structure of Fig. 2. We proceed to show that:

1) The field distributions of these fields can be iterated with the period $2z_t$ by performing appropriate phase transformations in the planes $z = z_t,\ 3z_t,\ 5z_t \cdots$;

2) The required phase transformations can be made the same for all partial fields by appropriately adjusting the mode parameters v_n and $\overline{v}_n$. All of these fields, regardless of their mode numbers and polarizations, are then iterated by one and the same guiding structure.

3) The partial fields satisfy orthogonality relations similar to the modes in conventional waveguides.

The fields (22) and (23) can thus be regarded as the modes of the dielectric slab-beam waveguide and, as in the case of conventional beam waveguides, may be called "beam modes".

To demonstrate these points we first note that the partial fields are conjugate complex in planes $+z =$ const and $-z =$ const. Hence the field distribution in the plane $z = -z_t$ can be reconstituted in the plane $z = +z_t$ by performing an appropriate phase transformation in this plane. The field distribution in the range $-z_t < z < +z_t$ is then repeated in the range $+z_t < z, +3z_t$. By iterating this process, i.e., by performing identical phase transformations in the planes $z = 3z_t, 5z_t \cdots$ the field distribution of the partial field is repeated periodically with the spacing $2z_t$ of the phase transformers. The required phase transformation $\Delta\phi$ is with (21):

$$\Delta\phi = \Delta\phi_n(y) = \frac{\frac{v_n^4}{\beta_n} z_t}{1 + \left(\frac{v_n^2}{\beta_n} z_t\right)^2} y^2. \tag{25}$$

It is quadratic in y and can be realized for example by a cylindrical lens. A lens of this type yields a phase transformation

$$\Delta\phi = -\hat{\phi}_n + \frac{1}{2}\frac{\beta_n}{f} y^2 \tag{26a}$$

which takes the desired form (25), if the focal length of the lens is chosen to be

$$f = \frac{z_t}{2}\left[1 + \left(\frac{\beta_n}{v_n^2}\frac{1}{z_t}\right)^2\right] \tag{26b}$$

$\hat{\phi}_n$ is a constant that depends on the shape of the lens.[1] Equations (21) to (23) show furthermore that with each iteration the partial fields are multiplied by a constant phase factor

$$\Gamma_{nm} = \exp\left\{-j\left[2\beta_n z_t + \hat{\phi}_n - (2m+1) \cdot \tan^{-1}\left(\frac{v_n^2}{\beta_n} z_t\right)\right]\right\} \tag{27}$$

which also includes the constant phase shift $\hat{\phi}_n$ of the lenses. Equations (25) to (27) apply to TM-fields; for TE-fields v_n, β_n are replaced by $\overline{v}_n$, $\overline{\beta}_n$.

The important point to observe is that the focal length (26) is independent of the mode number m and, in addition, becomes independent of the mode number n and the polarization (TE or TM) of the partial fields if

$$\frac{v_n^2}{\beta_n} = \frac{\overline{v}_n^2}{\overline{\beta}_n} = \text{const.} \neq f(n) \tag{28}$$

In other words, with this condition, all partial fields of arbitrary orders n, m and both polarizations are iterated by one and the same sequence of phase transformers. Conversely, a dielectric slab-beam waveguide with a given set of lenses of focal length f and spacing $2z_t$ will iterate all partial fields (22) and (23) provided their mode parameters are chosen according to the relations

$$v_n^2 = \frac{\beta_n}{\sqrt{z_t(2f - z_t)}} \quad \text{and} \quad \overline{v}_n^2 = \frac{\overline{\beta}_n}{\sqrt{z_t(2f - z_t)}} \tag{29}$$

which follow from (26b) and (28). Note that the propagation constants β_n, $\overline{\beta}_n$ are given quantities determined by the thickness d and permittivity ε_s of the dielectric slab, and that the focal length f must exceed $z_t/2$ for (29) to have real solutions, i.e. for iteration to occur.

[1]For the convex lenses of Fig. 1(a) we have $\hat{\phi}_n = (\beta_n/2f)y_0^2$, where $2y_0$ is the lateral width (aperture) of the lenses. For the concave lenses of Fig. 1(b), $\hat{\phi}_n$ is zero if the lens thickness is very small at the center.

The beammodes (22) and (23) satisfy the orthogonality relations

$$\int_{x=0}^{\infty}\int_{y=-\infty}^{\infty} E_x^{n,m} H_y^{*n',m'}\, dy\, dx = \sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{k_0}{\beta_n} \cdot \int_{x=0}^{\infty}\int_{y=-\infty}^{\infty}\left(\frac{k}{k_0}\right)^2 E_x^{n,m}E_x^{*n',m'}\, dy\, dx$$
$$= \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\beta_n}{k_0}\int_{x=0}^{\infty}\int_{y=-\infty}^{\infty}\left(\frac{k_0}{k}\right)^2 H_y^{n,m}H_y^{*n',m'}\, dy\, dx$$
$$= \sqrt{\frac{\varepsilon_0}{\mu_0}}\,\pi^{\frac{3}{2}} m!\,\frac{v_n}{\beta_n}\frac{1}{\varepsilon_s} \cdot\left[k_0 d + \frac{1}{\sqrt{\left(\frac{\beta_n}{k_0}\right)^2 - 1}\left[\left(\frac{\beta_n}{k_0}\right)^2(\varepsilon_s+1)-\varepsilon_s\right]}\right]\delta_{nn'}\,\delta_{mm'} \tag{30a}$$

for TM-modes, and

$$-\int_{x=0}^{\infty}\int_{y=-\infty}^{\infty} E_y^{n,m} H_x^{*n',m'}\, dy\, dx = \sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{\overline{\beta}_n}{k_0} \cdot \int_{x=0}^{\infty}\int_{y=-\infty}^{\infty} E_y^{n,m}E_y^{*n',m'}\, dy\, dx$$
$$= \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{k_0}{\overline{\beta}_n}\int_{x=0}^{\infty}\int_{y=-\infty}^{\infty} H_x^{n,m}H_x^{*n',m'}\, dy\, dx$$
$$= \sqrt{\frac{\varepsilon_0}{\mu_0}}\,\pi^{\frac{3}{2}} m!\,\frac{\overline{\beta}_n\overline{v}_n}{k_0^2}\left[k_0 d + \frac{1}{\sqrt{\left(\frac{\overline{\beta}_n}{k_0}\right)^2 - 1}}\right]\delta_{nn'}\,\delta_{mm'} \tag{30b}$$

for TE-modes. Asterisks indicate conjugate complex values. The mutual orthogonality of TM-modes versus TE-modes is obvious since (in the approximation used here) they do not have common transverse components. Equation (30) follows directly from (11) and (24).

With relations (30) any field guided by the dielectric slab-beam waveguide can be expanded into the beammodes of this guide. The expansion is complete provided that the field satisfies the wavebeam condition (15) with regard to its y-dependence and, concerning its x-dependence, behaves as a surface wave field of the dielectric slab.

While the field distribution of each beammode is strictly periodic with the spacing of the phase transformers, this does not necessarily apply for a composite wavebeam consisting of several beammodes. With each iteration the beammodes are multiplied by the phase factors $\Gamma_{n,m}$, (27), which depend on the mode numbers n and m, and the complex amplitude spectrum of the wavebeam will vary from section to section of the guide. The total power of this wavebeam, however, is preserved since the beammodes are powerwise orthogonal and the absolute value of each $\Gamma_{n,m}$ is unity. Note that this holds for the idealized dielectric slab-beam waveguide considered here having lossless phase transformers and infinite dimensions in the x- and y-directions. The iteration losses that occur in guides of finite cross section are discussed in Section III.

The beammodes (22), (23) have a constant beamwidth in the x-direction, but their beamwidth in the y-direction varies periodically with z. This beam width has a minimum halfway between the phase transformers and a maximum at the location of the lenses. With (21) and (29) the $1/e$-beamwidth at these positions is given by

$$\Delta w_{\min} = \frac{2}{v_n} = 2\left(\frac{z_t}{\beta_n}\right)^{\frac{1}{2}}\left(\frac{2f}{z_t} - 1\right)^{\frac{1}{4}} \tag{31a}$$

$$\Delta w_{\max} = \frac{2}{v_n}\left[1 + \left(v_n^2\frac{z_t}{\beta_n}\right)^2\right]^{\frac{1}{2}} = 2\left(\frac{z_t}{\beta_n}\right)^{\frac{1}{2}}\left[\frac{z_t}{2f}\left(1 - \frac{z_t}{2f}\right)\right]^{-\frac{1}{4}}. \tag{31b}$$

The expressions apply to the fundamental Gaussian mode of TM polarization. The corresponding formulas for TE polarization are obtained by replacing v_n, β_n with $\overline{v}_n$, $\overline{\beta}_n$. For higher order Gauss-Hermite modes the beamwidth will be somewhat larger. For given lens spacing $2z_t$, optimum beam confinement near the z-axis is achieved when the focal length f of the lenses is chosen such that $\Delta w_{\max}$ is as small as possible, which will occur for $f \simeq z_t$, i.e., in the "confocal" case that the focal points of adjacent lenses coincide. This result is well known from the theory of conventional beam waveguides.

A useful measure for the lateral extent of the beammodes in the x-direction is the fraction η_n of the total power of these modes that is transmitted in the air region outside the dielectric slab. With (7), (22), and (23), η_n and $\overline{\eta}_n$ are easily calculated:

$$\eta_n = \frac{\int_d^{\infty} F_n^2(x)\, dx}{\int_0^{\infty}\left(\frac{k_0}{k}\right)^2 F_n^2(x)\, dx} = \frac{\varepsilon_s}{\varepsilon_s - 1} \cdot \frac{\varepsilon_s - \left(\frac{\beta_n}{k_0}\right)^2}{k_0 d\left[\left(\frac{\beta_n}{k_0}\right)^2 - 1\right]^{\frac{1}{2}}\left[\left(\frac{\beta_n}{k_0}\right)^2(\varepsilon_s + 1) - \varepsilon_s\right] + \varepsilon_s}$$

for TM modes (32a)

$$\overline{\eta}_n = \frac{\int_d^{\infty} G_n^2(x)\, dx}{\int_0^{\infty} G_n^2(x)\, dx} = \frac{1}{\varepsilon_s - 1} \cdot \frac{\varepsilon_s - \left(\frac{\overline{\beta}_n}{k_0}\right)^2}{k_0 d\left[\left(\frac{\overline{\beta}_n}{k_0}\right)^2 - 1\right]^{\frac{1}{2}} + 1}$$

for TE modes. (32b)

Figs. 3 and 4 show η_n and $\overline{\eta}_n$ versus $k_o d$ for various slab permittivities. Away from cut-off, power in the air region is small, in particular for large ε_s, and most of the energy of the beammodes is transported inside the dielectric.

To avoid overmoding, it may often be desirable to chose the slab thickness d sufficiently small so that the guide supports only the $n = o$ group of Gauss–Hermite beam modes. The condition on d is:

$$0 < k_0 d < \frac{\pi}{\sqrt{\varepsilon_s - 1}} \quad \text{for TM modes} \tag{33a}$$

$$\frac{1}{2}\frac{\pi}{\sqrt{\varepsilon_s - 1}} < k_0 d < \frac{3}{2}\frac{\pi}{\sqrt{\varepsilon_s - 1}}. \quad \text{for TE modes} \tag{33b}$$

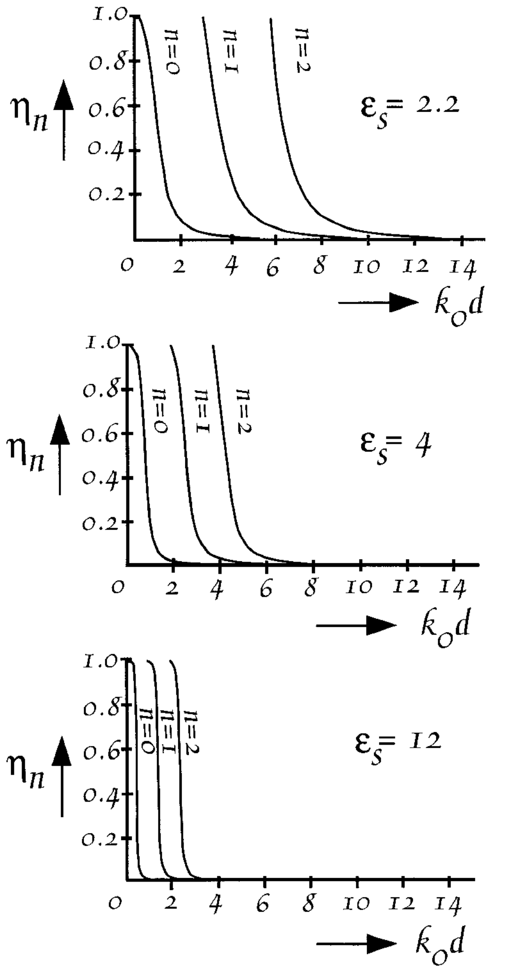


Fig. 3. Fraction η_n of power of TM polarized beammodes transmitted outside dielectric slab: η_n versus $k_0 d$ for various ε_s.

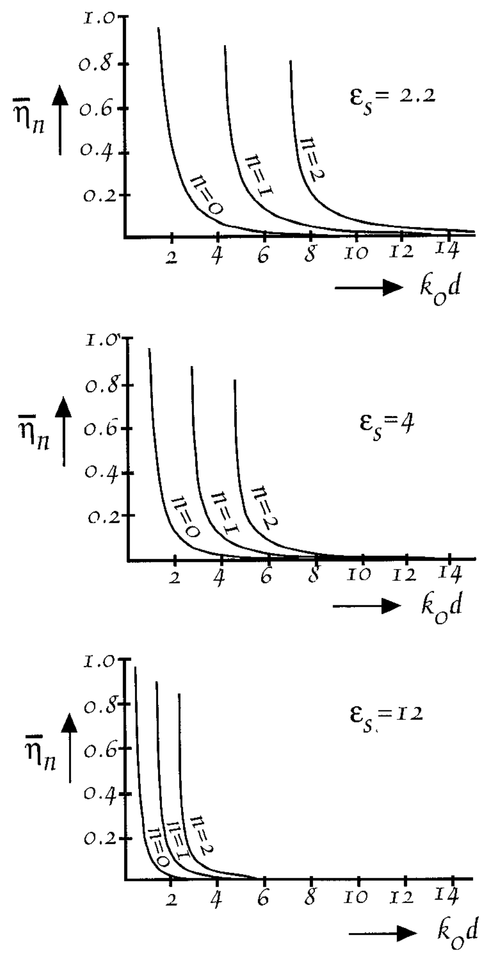


Fig. 4. Fraction $\overline{\eta}_n$ of power of TE polarized beammodes transmitted outside dielectric slab: $\overline{\eta}_n$ versus $k_0 d$ for various ε_s.

The upper limits are given by the appearance of the $n = 1$ group of beammodes. Near this upper limit the percentage of the power of the $n = 0$ beammodes transmitted outside the slab is small, i.e.

$$\begin{aligned} &\eta_0 = 0.028 \quad \overline{\eta}_0 = 0.061 \qquad \text{for} \quad \varepsilon_s = 2.2 \\ &\eta_0 = 0.018 \quad \overline{\eta}_0 = 0.060 \qquad \varepsilon_s = 4.0 \\ &\eta_0 = 0.0085 \quad \overline{\eta}_0 = 0.059 \qquad \varepsilon_s = 12. \end{aligned} \tag{34}$$

In this configuration, the dielectric slab-beam waveguide should be particularly well suited for the design of planar quasi-optical circuits.

The characteristics of the beammodes of the dielectric slab-beam waveguide may be summarized as follows:

1) In the direction normal to the slab surface (x-direction) the beammodes behave as surface waves guided by the slab. Their magnitude decreases exponentially away from the slab and their energy is largely confined to the interior of the slab.

2) In the lateral direction parallel to the slab surface (y-direction) the beammodes behave as reiterative wavebeams of the Gauss–Hermite type which are guided by the sequence of equally spaced identical phase transformers that are inserted in the slab and periodically reset the cross sectional phase distribution of the beammodes.

3) The propagation constant of the beammodes in the longitudinal direction (z-direction) equals k_0 at cut-off and, with increasing frequency, approaches k_s. It is always within the region $k_0 < \beta_n, \overline{\beta}_n < k_s$ thus characterizing the beammodes as surface waves guided by the dielectric slab.

4) The beammodes form a system of orthogonal modes that allows the complete description of any wavebeam guided by the dielectric slab-beam waveguide.

5) While conventional beam waveguides are virtually nondispersive if $z_t,\ f \gg \lambda_0$, the beammodes of the dielectric slab-beam waveguide show the dispersion of the dielectric slab guide. The phase velocity and group velocity of the beammodes can be derived from (8) and (27). Disregarding dispersion effects caused by the phase transformers (which should be small for thin lenses) one obtains

$$v_p \approx \frac{\omega}{\beta_n - \frac{m + \frac{1}{2}}{\sqrt{z_t(2f - z_t)}}} \approx \frac{\omega}{\beta_n} = \frac{c_0}{\sqrt{\varepsilon_{\text{eff}}}} \tag{35a}$$

$$v_g \approx \frac{\beta_n}{\omega} \frac{c_0^2}{\varepsilon_s(1 - \eta_n) + \eta_n} = \frac{c_0}{\sqrt{\varepsilon_{\text{eff}}}} \frac{\varepsilon_{\text{eff}}}{\varepsilon_{\text{avg}}} \tag{35b}$$

where c_o is the free space wave velocity, ε_{eff} is defined in the usual manner as $(\beta_n/k_o)^2$, and the "average" dielectric constant of the guide

$$\varepsilon_{\text{avg}} = \varepsilon_s(1 - \eta_n) + \eta_n$$

is obtained by weighting the permittivities of the dielectric slab and the air region with the relative powers transmitted in these regions. Equation (35) holds for both TM and TE polarization if in the latter case β_n, η_n are replaced by $\overline{\beta}_n$, $\overline{\eta}_n$.

A recent experimental study [3] has yielded results in good agreement with the theory presented here.

III. Dielectric Slab-Beam Waveguide of Finite Cross Section

The theory presented in Section II applies to an idealized dielectric slab-beam waveguide of infinite cross section dimensions. Since the beammodes (22), (23) decrease exponentially away from the z-axis in both transverse directions one may expect that the performance of the guide is not appreciably degraded in the actual case of finite dimensions.

The situation however is somewhat more complex for the following reason. In the case of a conventional beam waveguide the "spill-over" effect due to the finite size of the phase transformers will not lead to field distortions. Any energy, that after passing a given lens by-passes the following lens, will be radiated away from the guide and can be regarded as lost. In the case of the dielectric slab-beam waveguide this spill-over energy (more precisely, the part of this energy caused by the finite y-dimension of the lenses and traveling within the dielectric slab), will be reflected at the side walls of the slab and bounce back and forth between these walls, with little attenuation, in particular when the permittivity of the slab is high and its thickness is sufficiently far above cut-off. To minimize field distortions the reflection coefficient of the sidewalls must be controlled, for example by covering these walls with absorbing material or by replacing vertical walls with tapered transitions, as indicated in Fig. 5. The associated iteration loss can be kept small by choosing the width w of the slab sufficiently large e.g. $w > 3\Delta w_{\max}$; see (31b).

A second problem derives from the limited height of the lenses in the x-direction. For ease of fabrication, the lenses should not extend beyond the upper surface of the dielectric slab, and in an actual guide, the phase transformation (25) will be performed only within the dielectric slab but not in the air region above it. Since part of the power of the beammodes is transmitted in the air region, this truncation of the phase correction will lead to scattering, resulting in an increased iteration loss, and mode conversion, possibly causing field distortions.

An estimate of these effects is derived in the Appendix. The guide considered here satisfies conditions (33), so that it supports only the beammode group of order $n = 0$. The phase transformers are assumed to be very thin, planar devices.[2] A fundamental Gaussian beammode of unit amplitude is assumed incident upon the phase transformer in the plane $z = z_t$ and determines the field distribution in the input plane of the device. The field distribution in the output plane is obtained by applying the phase transformation (25) in the region $0 < x < d$, i.e. within the dielectric slab, while no phase correction is performed in the air region $d < x < \infty$.

[2] The width of the phase transformers in the y-direction is assumed to be sufficiently large in the present case, so that any iteration losses are caused by their finite height only.

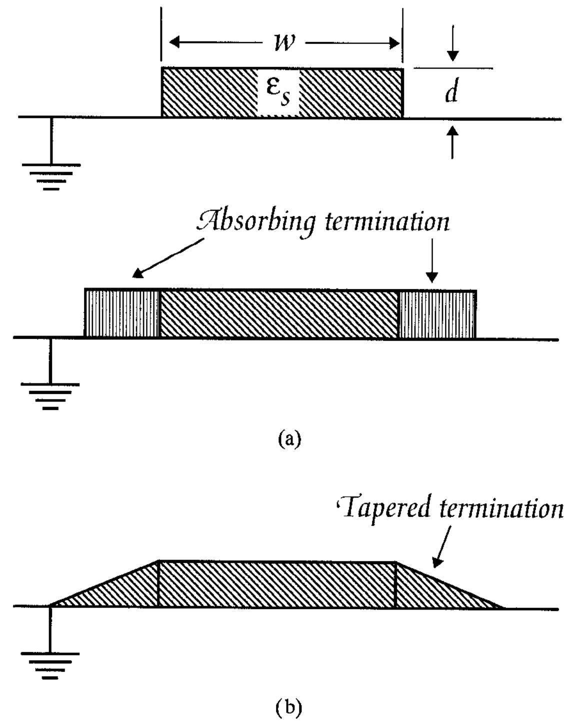


Fig. 5. Arrangements to suppress reflection at sidewalls of dielectric slab: (a) use of absorbing material. (b) use of slanted instead of vertical sidewalls.

Using the orthogonality relations (30), the field distribution in the output plane is expanded into the beammode spectrum of the guide section $z_t < z < 3z_t$. The power P_o of the fundamental Gaussian beammode will be smaller than that of the incident beammode, and is a measure for the iteration loss. The powers P_m in higher-order Gauss–Hermite beammodes indicate the magnitude of the mode conversion effect. The power P_s scattered by the truncated phase transformer is found by invoking energy conservation, i.e., by subtracting the power of the combined beammode spectrum of the guide section $z_t < z < 3z_t$ from the power of the incident beammode (see discussion in the Appendix).

For a conformal guide with $f = z_t$ one obtains:

$$\frac{P_0}{P_{\text{inc}}} = 1 - 0.45\eta_0 + 0.15\eta_0^2 \tag{36a}$$

$$\sum_{m=2,4\cdots}^{\infty} \frac{P_m}{P_{\text{inc}}} = 0.30\eta_0^2 \tag{36b}$$

$$\frac{P_s}{P_{\text{inc}}} = 1 - \sum_{m=0}^{\infty} \frac{P_m}{P_{\text{inc}}} = 0.45\eta_0(1 - \eta_0) \tag{36c}$$

where P_{inc} is the power of the incident Gaussian beammode and η_0 is given by (32). The formulas hold for both TM- and TE-polarization where in the TE case η_0 is replaced by $\overline{\eta}_0$.

Equations (36) indicate that roughly one half of the power transmitted outside the dielectric slab is lost with each iteration. Most of this power is scattered away from the guide, while the power transformed into higher order beammodes is proportional to η_0^2 and small in higher order. Hence little mode conversion will occur when η_0 is small, i.e. when the electrical thickness of the slab, $k_o d$, is sufficiently far above cut-off; see (34). In this region of small η_0, the iteration loss is expected to be in the order of a few percent, depending on the guide permittivity, while field distortions will be minimal.

The total iteration loss of an actual dielectric slab-beam waveguide, of course, consists of several parts including dielectric losses in the slab material; reflection and absorption losses of the lenses; and diffraction losses due to the finite size of the lenses both in the x- and y-directions. All of these losses can be made small, by appropriate design of the guide, except for the loss associated with the finite height of the lenses, which is inherent with the guide configuration. Equation (36) provides a first estimate of this loss; an accurate determination will require further study.

Appendix

Estimate of Effects of Finite Height of Phase Transformers

In an actual dielectric slab-beam waveguide, the height of the phase transforming lenses will not extend beyond the dielectric slab; see Fig. 1. This height limitation has two effects:

1) Some of the energy of the beammodes will be scattered away from the guide, leading to an iteration loss.

2) Coupling between the beammodes will occur, leading to mode conversion and field distortions.

We estimate these effects for a guide whose electrical thickness $k_o d$ satisfies conditions (33) so that the guide supports only the $n = 0$ group of beammodes. We assume furthermore that the phase transformers are thin, planar devices and that their width w is sufficiently large so that their finite y-dimension does not noticeably contribute to the iteration loss. The calculations below apply to the TM case but are analogous in the TE case.

A fundamental Gaussian beammode of unit amplitude is incident upon the phase transformer at $z = z_t$ in Fig. 2. With (22a) its field distribution in the input plane of the phase transformer is given by:

$$E_x = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\frac{k_0\beta_0}{k^2}\,H_y = \left(\frac{k_0}{k}\right)^2 F_0(x)Q_{00}(x, z_t)e^{-j\beta_0 z_t}$$
$$\text{for } z = z_t - \delta,\ \delta \to 0$$
$$\text{with } k = k_s \quad \text{for } x < d$$
$$k = k_0 \quad \text{for } x > d. \tag{37}$$

The transverse field components only are shown here since they determine the field uniquely. The field distribution in the output plane is obtained by applying the phase transformation (25)

$$\Delta\phi(y) = -\hat{\phi} + \frac{\frac{v_0^4}{\beta_0} z_t}{1 + \left(\frac{v_0^2}{\beta_0} z_t\right)^2} y^2$$

in the slab region $0 < x < d$, while no phase correction is performed in the air region $d < x < \infty$. Thus

$$E_x = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\frac{k_0\beta_0}{k_s^2}\,H_y = \frac{1}{\varepsilon_s} F_0(x)Q_{00}(y, z_t)$$
$$\cdot e^{-j[\beta_0 z_t - \Delta\phi(y)]} \quad 0 < x < d$$
$$E_x = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\frac{\beta_0}{k_0}\,H_y = F_0(x)Q_{00}(y, z_t)e^{-j\beta_0 z_t}$$
$$d < x < \infty$$
$$\text{for } z = z_t + \delta,\ \delta \to 0 \tag{38}$$

The field in the space range $z_t < z < 3z_t$ is written as a superposition of beammodes

$$E_x = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\frac{k_0\beta_0}{k^2}\,H_y$$
$$= \left(\frac{k_0}{k}\right)^2 F_0(x) \sum_{m=0}^{\infty} \cdot A_m(\Gamma_{00}\Gamma_{0m})^{\frac{1}{2}}$$
$$\cdot Q_{0m}(y, z - 2z_t)e^{-j\beta_0(z-2z_t)}$$
$$\text{for} \quad z_t < z < 3z_t \tag{39}$$

where Γ_{nm} is given by (27). The expansion coefficients A_m are obtained by equating the fields (38) and (39) in the plane $z = z_t + \delta$ and application of the orthogonality relation (30a). The necessary integrations can be performed analytically with the result

$$A_0 = 1 - \left(1 - \frac{e^{j\hat{\phi}}}{\sqrt{1 + jw_t}}\right)\eta_0 \qquad m = 0 \tag{40a}$$

$$A_m = \frac{(-j)^{\frac{m}{2}}}{2^{\frac{m}{2}}\left(\frac{m}{2}\right)!}\,\frac{e^{j\hat{\phi}}}{\sqrt{1 + jw_t}}\left(\frac{w_t}{1 + jw_t}\right)^{\frac{m}{2}}\eta_0$$
$$m\text{:even} \tag{40b}$$

$$A_m = 0 \qquad m\text{:odd}$$

with

$$w_t = \frac{v_0^2}{\beta_0} z_t = \left(\frac{z_t}{2f - z_t}\right)^{\frac{1}{2}}.$$

Note that the beammode expansion (39), (40) represents only the *guided* field in the space range $z_t < z < 3z_t$, i.e. the field that can be expressed in terms of the surface wave modes of the dielectric slab. The field *scattered* at the (truncated) phase transformers would have to be expressed in terms of the radiative modes of this guide. These modes have been disregarded in the main text; but their total power can be obtained by invoking energy conservation. The radiative modes [6], [7] are powerwise orthogonal to the guided modes (beammodes) and we have

$$P_s = P_{\text{inc}} - \sum_{m=0}^{\infty} P_m \tag{41}$$

where P_{inc} is the power of the incident wavebeam, P_m is the power of the beammode of order m and P_s is the scattered power. With (30a) and (39) through (40)

$$\frac{P_0}{P_{\text{inc}}} = \left|1 - \eta_0\left(1 - \frac{e^{j\hat{\phi}}}{\sqrt{1 + jw_t}}\right)\right|^2 \tag{42a}$$

$$\sum_{m=2,4\cdots}^{\infty} \frac{P_m}{P_{\text{inc}}} = \eta_0^2\left(1 - \frac{1}{\sqrt{1+w_t}}\right) \tag{42b}$$

$$\frac{P_s}{P_{\text{inc}}} = 2\eta_0(1-\eta_0)\left[1 - \text{Re}\left(\frac{e^{j\hat{\phi}}}{\sqrt{1+jw_t}}\right)\right] \tag{42c}$$

where η_0, (32a), is the fraction of the total power of each beammode that is transmitted outside the dielectric slab. Away from cut-off, η_0 is small; see Figs. 3 and 4. Equation (42a) shows the relative power of the fundamental beammode after passing the phase transformer and indicates the iteration loss. Equation (42b) is the power converted into higher order Gauss–Hermite beam modes and indicates the magnitude of field distortion effects; due to the symmetry of the problem, only the beam modes of even order are excited.

The expressions for P_0, and P_s depend significantly on $\hat{\phi}$, the electrical thickness of the phase transformers at their center $y = 0$. The primary reason for this dependence is the assumption of planar, infinitely thin phase transformers which cause a sharp phase jump (field discontinuity) at their upper edge $x = d$. Actual phase transformers (lenses) have a center thickness of several wavelengths or more, and the phase jump is smoothed out into a gradual phase transition. To simulate this situation we assume in the following $\hat{\phi} = 2\mu\pi$ (μ: integer) so that the phasejump is zero at $y = 0$ where the field strength of the incident mode is at a maximum. Assuming furthermore a confocal guide with $f = z_t$ and $w_t = 1$, (42) reduce to

$$\frac{P_0}{P_{\text{inc}}} = 1 - 0.45\eta_0 + 0.15\eta_0^2 \tag{43a}$$

$$\sum_{m=2,4\cdots}^{\infty} \frac{P_m}{P_{\text{inc}}} = 0.30\eta_0^2 \tag{43b}$$

$$\frac{P_s}{P_{\text{inc}}} = 0.45\eta_0(1-\eta_0) \tag{43c}$$

indicating that roughly 50% of the power transmitted outside the dielectric slab is lost with each iteration. Most of this power is scattered away from the guide while the power converted into higher order modes is proportional to η_0^2 and a second order effect.

Equations (42) and (43) were derived for the TM-case but apply equally to the TE-case if η_0 is replaced by $\overline{\eta}_0$. The equations should be regarded as a first estimate; an accurate assessment of the iteration loss will require further study.

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F. K. Schwering (M'60–SM'86–F'88), for photograph and biography see this issue, p. 1826.